APPENDIX I
ACTIVITY #4 – ECCENTRICITY OF ORBIT

Overview: The Equation of Time (“EoT”) curve from Activity #3 is a composite of two effects: (1) the tilt of the Earth’s axis, and (2) the eccentricity of Earth’s orbit. In Activity #4, an EoT curve due solely to the tilt of the Earth’s axis is generated, and subtracted from the composite EoT curve from Activity #3 … the result is an EoT curve due solely to the eccentricity of Earth’s orbit, and it is constructed solely from data in the experimental analemma.

A second EoT curve due solely to the eccentricity of Earth’s orbit is generated from theoretical considerations and compared to the experimental result on a point-by-point basis. The eccentricity is determined by trial-and-error using a least-squares technique. (i.e., the reported eccentricity is that value of eccentricity that results in the least sum of the squares of the difference between the experimental and theoretical analemma.)

Observers are expected to utilize all points of their analemma. Techniques which utilize analemma points at selected positions in Earth’s orbits will not be accepted.

Acknowledgements: The approach taken here is from two sources:
www.analemma.com
Practical Astronomy With Your Calculator , by Peter Duffett-Smith

Observers are encouraged to review these sources.

Step 1: Calculate the EoT for an Earth with a tilted axis in a circular orbit (eccentricity = 0):

- Let α = Tilt of Earth’s axis (from Activity #1).
- Let N = Day Number (Jan 1st = 1 ; typical Vernal Equinox = Mar 21st = 80).
- Let ε = the angle of the mean Sun after Vernal Equinox.

$$\varepsilon = 360 * (N - 80) / 365.24.$$

if $\varepsilon >= 270°$, subtract 360° from ε.
if $\varepsilon >= 90°$, subtract 180° from ε.

- Let β = the angle of the true Sun.

$$\beta = \arctan(\cos(\alpha) * \tan(\varepsilon)).$$

- EoT = $(\varepsilon - \beta)$ * (1440 clock minutes / 361 degrees of rotation).

Step 2: Subtract the EoT calculated in Step 1 above from the composite EoT calculated in Activity #3. The result is an EoT for an untilted Earth in an elliptical orbit which is determined solely by data from the analemma.

Step 3(a): Calculate the EoT for an untilted Earth in an elliptical orbit using geometric and trigonometric considerations (i.e., “theory”):

- Let N = Day Number. (Jan 1^{st} = 1; typical Perihelion = Jan 2^{nd} = 2)
- Let λ = the orbital angle of the mean Sun after Perihelion.

  λ = (360° of orbit in one year) * (N – 2) / (365.24 days in one year).

- $\nu = \lambda + (360 / \pi) * e * \sin(\lambda)$.

  This is an abbreviated version of the **Equation of the Center**. It is applicable ***only*** to orbits with very low eccentricity … in addition; both λ and ν are expressed in degrees.

  e = the trial-and-error value of orbital eccentricity.

- EoT = (λ – ν) * (1440 clock minutes / 361 degrees of rotation).

Step 3(b): Subtract the EoT determined in Step 3(a) (“theory”) from the EoT determined in Step 2 (“experimental”); square the difference; sum these squares for all of the analemma points.

Step 3[c]: Adjust the value of eccentricity (“e”) used in Step 3(a) such that the sum of the squares gets smaller. (i.e., the fit between experimental and theoretical values gets better.) Report the value of “e” which produces the minimum of the sum of the squares in Step 3(b). “e” should be carried out produced to the fourth significant digit (should be the fifth decimal digit).