# APPENDIX G
# ACTIVITY #2 – THE PATH OF THE SUN IN THE SKY

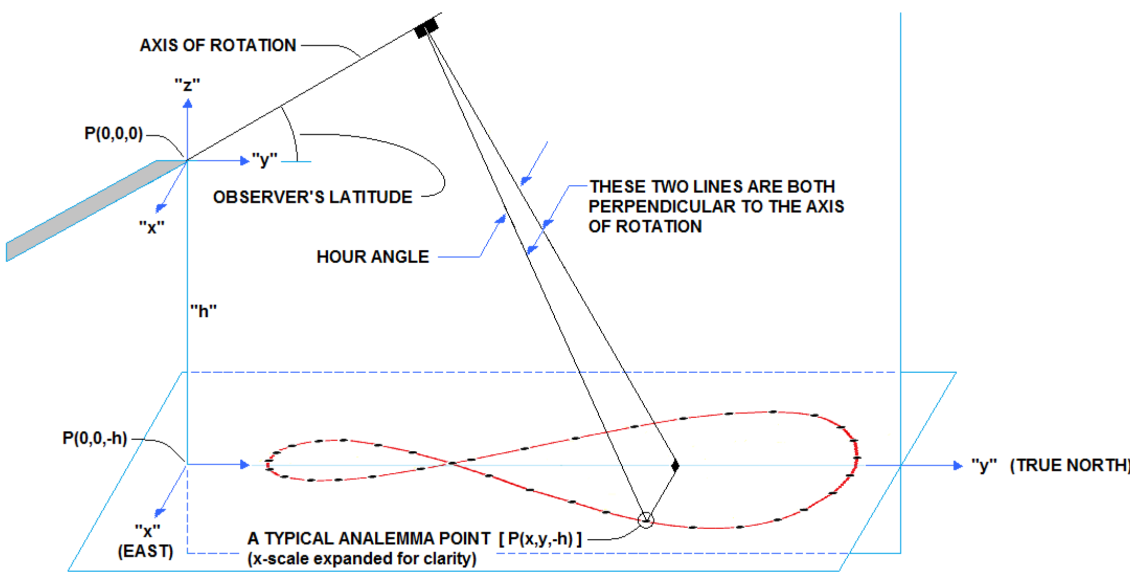
Though several options are available to portray the path of the Sun in the Sky, most will find it easier overall to:

- First, calculate the altitude and azimuth of the Sun at each point of the analemma (Steps 1 through 4, below).
- Second, using the equations for conversion of alt-azimuth coordinates to equatorial coordinates, convert the alt-azimuth coordinates to Declination and Hour Angle (Step 5, below).

**Step 1:** Continue with the coordinate system introduced in Activity #1:

- P(0,0,0) at the opening of the enclosure / tip of the gnomon.
- The x-axis as east / west (positive being eastward).
- The y-axis as north / south (positive being northward).
- The z-axis as up / down (positive being upward).

Note that the analemma is in the x / y plane at z = -h.




**Step 2:** Digitize the analemma. This will generally be done by scanning or photographing the analemma. Be sure to allow for the following:

- The location of the point directly below the opening of the enclosure / tip of the gnomon … this point will be referred to as P(0,0,-h) . Accurate measurement of distances relative to this point is crucial to the calculations which follow.
- If photographing the analemma, take the image from directly above (i.e., perpendicular to) the analemma using as long a focal length as possible

to minimize distortion.Wide-angle lenses should not be used. The "long dimension" of the analemma should be aligned with the width of the camera's field of view as well as possible.

- Scaling the scan / image must be included, so conversion from locations in the image to measured distances can be made.

**Step 3:** Translate the zero-point on the image from Step 2 (generally the upper left corner) to P(0,0,-h) … i.e., the point directly below the opening in the enclosure / tip of the gnomon in the coordinate system described in Step 1.

When translating the origin of a coordinate system to a point having the coordinates P(h,k) within that system, then the coordinates of a point P(x,y) will change to:

$$x' = x - h$$
$$y' = y - k$$

where: x and y refer to the original (pre-translation) coordinates.
x' and y' refer to the post-translation coordinates.

Note: This relationship assumes an x-positive to the right / y-positive up orientation. If the image's coordinate system has different orientation, corrective measures will have to be taken.

**Step 4:** For each point of the analemma, calculate the altitude-azimuth coordinates:

- Calculate new x and y values based on translation of the axes to the point on the floor of the enclosure directly below the opening ( P(0,0,-h) ).
- Provide for calculation new x and y values based on the rotation of the axes around the z-axis. (This is a correction for magnetic deviation, improper alignment of the observing apparatus along true north / south, or if photographing the analemma, not properly aligning the analemma within the camera's field of view.) Initially, this angle of rotation will be set to 0° (i.e., not rotated).

When rotating a coordinate system an angle $\alpha$ (alpha) around its origin,

$$X = x * \cos(\alpha) + y * \sin(\alpha)$$
$$Y = -x * \sin(\alpha) + y * \cos(\alpha)$$

where: x and y refer to the original (pre-rotation) coordinates.
X and Y refer to the post-rotation coordinates.

Note: $\alpha$ is positive in the counter-clockwise direction.

- Calculate the angle off the x=0 plane (a.k.a., the y /,z plane, which contains the Celestial Meridian). Numerically, it is the arctan(x / y); above the opening, it is the angle, $\phi$ (phi). Note that the Sun's Azimuth is 180° + $\phi$.
- Calculate the angle off the x, y ("horizontal") plane. Numerically, it is arctan( h/sqrt($x^2$+$y^2$) ); above the opening, it is the angle, $\theta$ (theta). Note that this is also the Sun's Altitude.

**Step 5:** Calculate the Declination and Hour Angle for the Sun at each reading in the analemma. The equations are presented in *Practical Astronomy With Your Calculator*, by Peter-Duffett Smith §26.

$$\sin(\delta) = \sin(a)*\sin(\varphi) + \cos(a)*\cos(\varphi)*\cos(A)$$

$$\cos(H) = ( \sin(a) - \sin(\varphi)*\sin(\delta) ) / ( \cos(\varphi)*\cos(\delta) )$$

where, a = altitude of the Sun (from Step 4, above).
A = Azimuth of the Sun (from Step 4, above).
$\delta$ = declination of the Sun.
$\varphi$ = Latitude of the Observer (from Activity #1).
H = the Hour-Angle between the Sun and the Meridian.

In the above equation for cos(H), the values of H will always be positive. To determine the sign of H use the following equation:

$$\sin(H) = - \sin(A) \cos(a) / \cos(\delta)$$

For a good description of how the above equations are derived see the following link: http://star-www.st-and.ac.uk/~fv/webnotes/chapter7.htm

**Step 6:** Plot the Declination (vertical axis) vs. Hour Angle (horizontal axis).

## Additional Links

Printable Version of this page
Return to Introduction page
Appendix A – Construction of an Enclosure
Appendix B – Overview of Non-Local Noon Observing
Appendix C – Unattended Photography
Appendix D – * * * Reserved for Future Use * * *
Appendix E – * * * Reserved for Future Use * * *
Appendix F – Activity #1 (Tilt of Earth's Axis and Observer's Latitude)
Appendix G – Activity #2 (Path of the Sun in the Sky)
Appendix H – Activity #3 (Equation of Time)
Appendix I – Activity #4 (Eccentricity of Orbit)