APPENDIX F
ACTIVITY #1 - CALCULATION OF THE TILT OF THE EARTH'S AXIS AND THE OBSERVER'S LATITUDE

The calculation of the Tilt of the Earth's Axis and the Observer's Latitude is essentially a reversal of the calculation sequence shown for the construction of the enclosure (Appendix A). In the discussion below, "tip of the gnomon" can usually be read in lieu of "opening of the enclosure" for participants who use a gnomon to cast a shadow instead of an enclosure with an opening to pass a light ray.

Step 1: setup a coordinate system with:

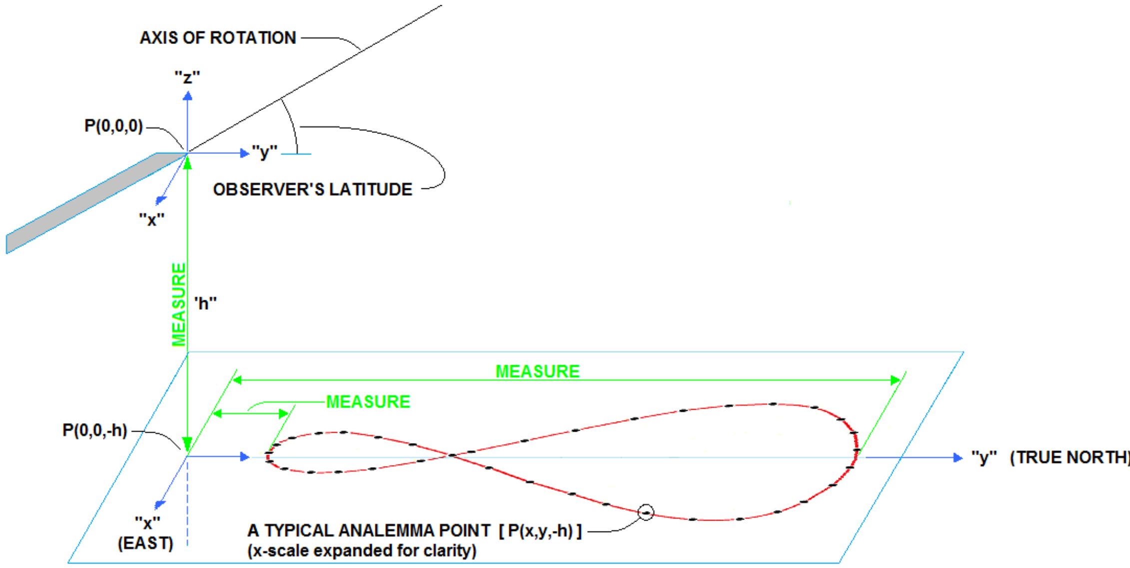
- P(0,0,0) at the opening of the enclosure / tip of the gnomon.
- the x-axis as east / west (positive being eastward).
- the y-axis as north / south (positive being northward).
- the z-axis as up / down (positive being upward).

For observers in the Southern Hemisphere, rotate the coordinate system 180 degrees around the Z axis, so positive Y is South and positive X is West.
Note: that the analemma is in the x / y plane at z = -h.

Step 2: Locate the point both directly below the opening in the enclosure and in the plane containing the analemma … this point will be referred to as "P(0,0,-h)." For the construction described in Appendix A, a heavy sewing needle works well when suspended through the opening by a thread on a windless day with the enclosure in its observing position.

Step 3: Make the following three measurements from P(0,0,-h) (determined in Step-1 above):

- Distance from the opening in the enclosure to the plane in which the analemma lies; in Appendix A, this distance is referred to as "h".
- Distance to the Summer Solstice point. (This is the point on the analemma curve closest to P(0,0,-h).)
- Distance to the Winter Solstice point. (This is the point on the analemma curve farthest from P(0,0,-h).)



For observers in the Southern Hemisphere relabel the figure such that positive Y is true South and positive X is West.

Step 4: Using the measurements above, calculate the altitude of the Sun at both Summer and Winter Solstices:

- (AltSummerSolstice) = arctan(h / (DistanceToSummerSolsticePoint))
- (AltWinterSolstice) = arctan(h / (DistanceToWinterSolsticePoint))

Note: for Northern Hemisphere observers, both altitudes are measured with respect to the Southern horizon. For Southern hemisphere observers, both altitudes are measured with respect to the Northern horizon.

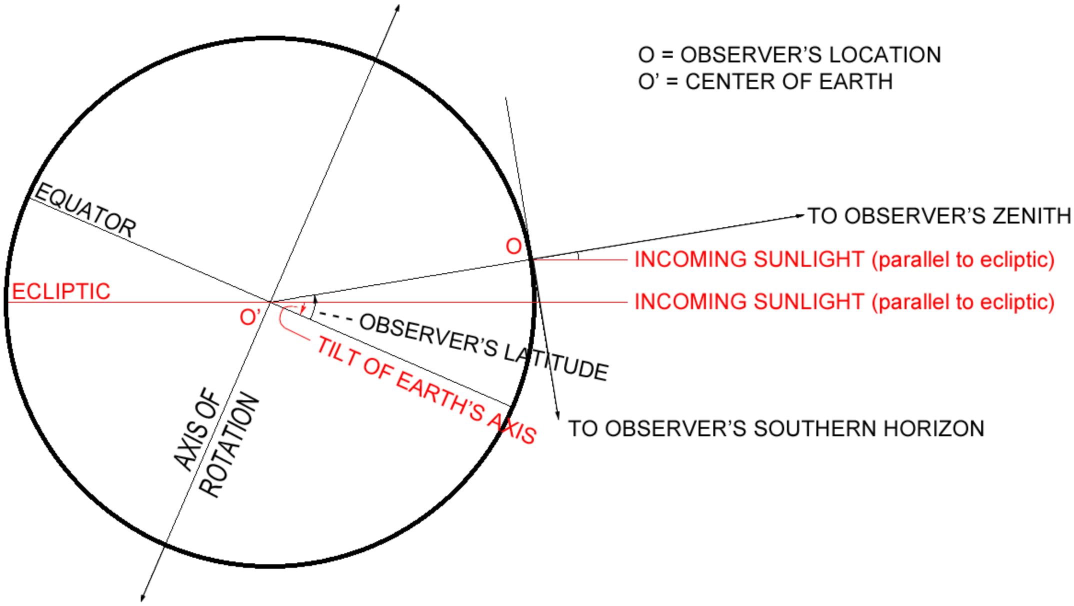
Step 5: Specify the relationships between the altitude of the Sun at Summer Solstice, the tilt of the Earth's axis, and the Observer's Latitude. With reference to the sketch below, specifically the 90° angle between the Observer's southern horizon and the Observer's zenith:

(AltSummerSolstice) + (ObsLatitude) – (TiltOfAxis) = 90°

The altitude of the Sun at Summer Solstice (AltSummerSolstice) is calculated from the dimensions of the analemma and the observing apparatus (Step 3).

Rearranging the equation gives:

(ObsLatitude) – (TiltOfAxis) = 90° - (AltSummerSolstice)



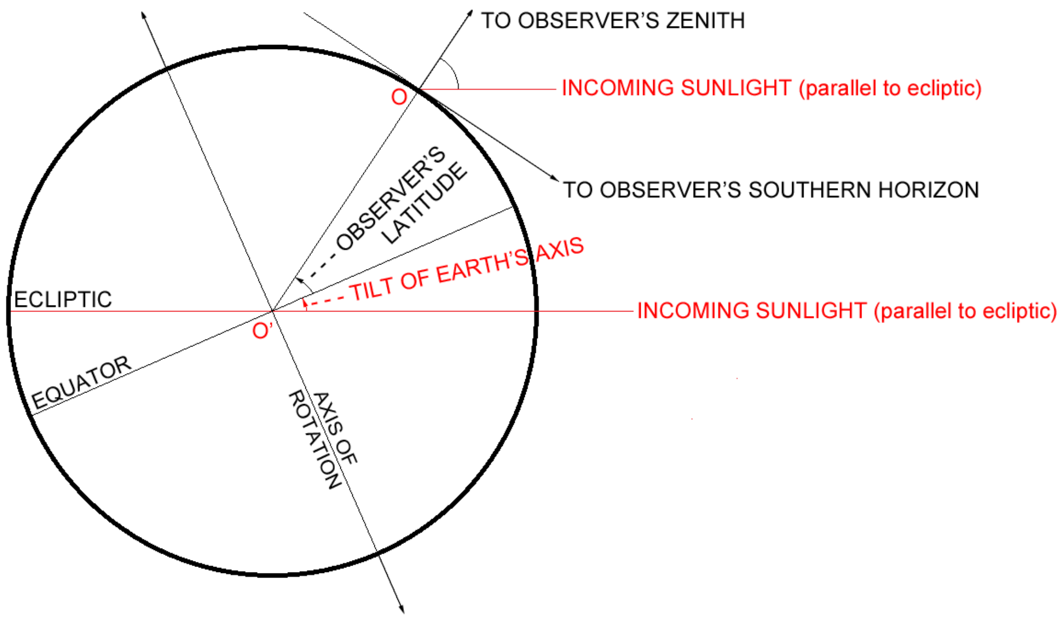
Step 6: Specify the relationships between the altitude of the Sun at Winter Solstice, the tilt of the Earth's axis, and the Observer's Latitude. With reference to the sketch below, specifically the 90° angle between the Observer's southern horizon and the Observer's zenith:

$$(\text{AltWinterSolstice}) + (\text{ObsLatitude}) + (\text{TiltOfAxis}) = 90°$$

The altitude of the Sun at Winter Solstice (AltWinterSolstice) is calculated from the dimensions of the analemma and the observing apparatus (Step 3).

Rearranging the equation gives:

$$(\text{ObsLatitude}) + (\text{TiltOfAxis}) = 90° - (\text{AltWinterSolstice})$$



Step 7: Specify the Observing Latitude by adding the equations from Step 4 and Step 5, above:

(ObsLatitude) – (TiltOfAxis)] + [(ObsLatitude) + (TiltOfAxis) =
90° - (AltSummerSolstice) + 90° - (AltWinterSolstice)

Rearranging the equation gives:

2*(ObsLatitude) = 2*90° - AltSummerSolstice) + (AltWinterSolstice)

Again, rearranging the equation gives:

(ObsLatitude) = 90° - ((AltSummerSolstice) + (AltWinterSolstice)) / 2

Use the solar altitudes calculated in Step 3, above.

Note: for Southern Hemisphere observers, the 'ObsLatitude' obtained from the above equation will be positive. This positive value should be used for the rest of the program. When final results are summarized, indicate South. Example 32.5 degrees South.

Step 8: specify the Tilt of the Earth's Axis by subtracting the equation from Step 4, above, from the equation from Step 5, above:

(ObsLatitude) + (TiltOfAxis) – (ObsLatitude) – (TiltOfAxis) =
90° - (AltWinterSolstice) – 90° - (AltSummerSolstice)

Rearranging the equation gives:

$$2*(TiltOfAxis) = (AltSummerSolstice) – (AltWinterSolstice)$$

Again, rearranging the equation gives:

$$(TiltOfAxis) = ((AltSummerSolstice) – (AltWinterSolstice)) / 2$$

Use the solar altitudes calculated in Step 3, above.